Clarification on The Peirce Translation
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1 Clarification of the Proof of Theorem 3.2

Theorem 3.2. *Assume that $P(x, y) \to R$, and that the variable y is not free in R. If*

$$\mathsf{ML} + J_R\text{-elim} \vdash \forall x \exists y P(x, y)$$

then also

$$\mathsf{ML} \vdash \forall x \exists y P(x, y).$$

Proof. First notice that $P(x, y) \to R$ implies that $\exists y P(x, y) \to R$, since y is not free in R. The J_R-elimination follows from the $J_{\exists y P(x,y)}$-elimination principle, since

$$(((A \to R) \to A) \to A$$

is monotone in R. Therefore, from

$$\mathsf{ML} + J_R\text{-elim} \vdash \forall x \exists y P(x, y)$$

it follows that

$$\mathsf{ML} + J_{\exists y P(x,y)}\text{-elim} \vdash \exists y P(x, y).$$

Now, it is easy to check that

(i) $J_{\exists y P(x,y)} P(x, y) \to P(x, y)$,

(ii) $J_{\exists y P(x,y)} \exists y P(x, y) \to \exists y P(x, y)$.

We also have that

$$(J_{\exists y P(x,y)} A)^{J_{\exists y P(x,y)}} \leftrightarrow J_{\exists y P(x,y)} (A)^{J_{\exists y P(x,y)}}.$$

Hence, Lemma 3.1 gives $\mathsf{ML} \vdash J_{\exists y P(x,y)} \exists y J_{\exists y P(x,y)} P(x, y)$ (taking the monad TA to be $J_{\exists y P(x,y)} A$) which by (i) and (ii), implies the conclusion. $\square$

The proof of Corollary 3.4 can be elaborated in a similar way.

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