BDP$_\forall$:
$\forall p\colon X \to 2 \;\; \exists a \in X(pa = 0 \implies \forall x \in X(px = 0))$.

BDP$_\exists$:
$\forall p\colon X \to 2 \;\; \exists a \in X(\exists x \in X(px = 1)) \implies pa = 1$.

searchable:
$\forall p\colon X \to 2 \;\; \exists a \in X\,(\neg\neg\exists x \in X(px = 1)) \implies pa = 1$.

PO:
$\forall p\colon X \to 2 \;\; (\exists x \in X(px = 1)) \vee (\forall x \in X(px = 0))$.

Dubuc-Penon compact:
$\forall A\colon \Omega \;\; \forall B\colon X \to \Omega \;\; (\forall x \in X(A \vee B(x))) \implies A \vee \forall x \in X(B(x))$.

BDP-compact:
$\forall A\colon \Omega \;\; \forall B\colon X \to 2 \;\; (\forall x \in X(A \vee B(x))) \implies A \vee \forall x \in X(B(x))$.